

GEARS WEIGHT EQUATIONS - GEAR CHAIN WEIGHT CALCULATION METHODOLOGY

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Abstract: Weight is a critical factor in aircraft engine design, where it is required to keep the smallest weight at the same strength properties. Minimum weight requirements are especially applicable at reduction gearboxes of the turbo propeller engines where high reduction ratio is required to transmit power from power turbine to the propeller. Depending on the required gear reduction value and available space different gears arrangements are used. Specific gear arrangements allow to transmit high reduction values at certain weight of the whole gear chain. The presented in the article methodology allows to size the gears and ratios of the gear chains, while keeping the lightest possible design, still meeting transmitted power and strength properties requirements. The presented methodology is applicable at the beginning of design process to judge which gear system will fit the best to specific performance requirements. Gear sizing outputs data from the preliminary gear chain sizing are the inputs data for the detail design. Methodology is applicable to spur, helical and planetary gears. Primary criteria to size gear dimension is maximum allowable surface durability factor. Based on the derived formulas there have been created weight graphs. Graphs show the relationship between gear ratio and weight for different types of gears arrangements and allow to choose specific type of gear system based on the required ratio.

Keywords: GEAR SIZING, GEAR WEIGHT

1. Introduction

Aircraft engine transmissions systems are assemblies from which high reliability and low mass is required. Weight limitations are related to gearboxes which are used to transmit power and torque. Gearboxes are one of the heaviest assemblies included in power transmission systems, that's why mass of the power train is one of the key factor which has an influence on gear train arrangement.

The article presents the methodology of gear chain mass sizing depending on kinematic drive scheme. Presented below methodology allows to size raw dimensions of the gears with respect to required gear ratio, transmitted power and gears contact strength properties. Experience of authors in gearbox design indicates that contact stress in gear mesh limits load carrying capacity of the gearbox.

2. Discussion

Gearboxes used in aircraft engines are built from a few typical schemes which are presented in figure 1, in example: offset gears (a), offset gears with idler (b), offset gears with two idlers (c), double reduction gears (d), multi branch gears (e) and planetary gears (f).

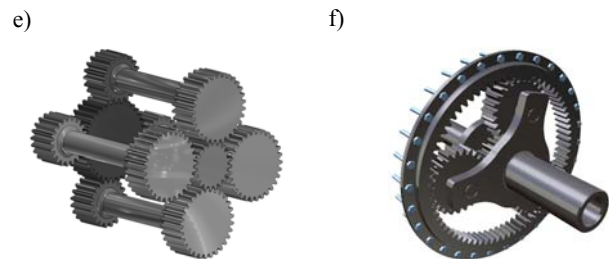
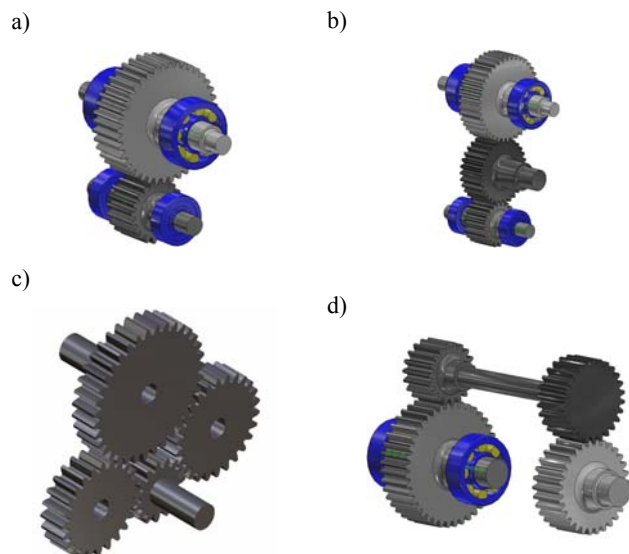


Fig. 1 Typical gears arrangement used in aircraft engine power transmission

Gearbox scheme depends on the role which it plays in transmission chain and required transmitted power. Strength and mass requirements are established by AGMA norm [1]. Design equations are based on the assumption that the weight of a gear drive is proportional to the solid rotor volume of the individual gears in the drive, $b * d^2 * \psi$, where:

b - width of the gear, d - pitch diameter, ψ – gear volume fill factor.

Gear volume fill factor is defined:

$$\psi = 1 - \frac{V_s}{V_p} \quad (1)$$

where: V_p – solid gear volume, V_s – gear material volume after final machining.

In gearboxes used in aircraft industry gear volume fill factor ψ value is from 0.3 to 0.7. When know contact stress relationship it is possible to establish required gears volume, necessary to transmit required load at known gearbox ratio. Relationship between geometrical dimensions of the gearbox, ratios and power shows equation [2]:

$$a^2 * b = 4,84 * 10^6 \frac{P * (i + 1)^3}{K * n_1 * i} \quad (2)$$

where: $a[mm]$ – axis to axis center distance of the gearbox stage,

$n_1 [1/min]$ – input gear speed i – gearbox ratio ; $i = \frac{n_1}{n_2} = \frac{d_2}{d_1}$

$P[kW]$ – power on input drive, K – surface durability factor.

Experience in gearbox design shows that maximum allowable surface durability factor should be $K=1,5-4$ MPa [3][4]. Center distance of mating gears describes relationship:

$$a = 0.5 * (d_1 + d_2) = 0.5 * d_1 * (1 + i) \quad (3)$$

After substitution equation (3) into (2) we have:

$$b * d_1^2 = 10,62 * 10^6 \frac{P}{K * n_1} * \frac{i+1}{i} \quad (4)$$

If power P is written as a function of the torque T

$$P = \frac{T_1 * n_1}{9500} [kW] \quad (5)$$

where: $T_1 [Nm]$ – torque on input gear.

then equation (4) will be written :

$$b * d_1^2 = 2000 * \frac{T_1}{K} * \frac{i+1}{i} \quad (6)$$

Real gear volume V_{r1} describes the equation:

$$V_{r1} = \frac{\pi * d_1^2}{4} * b * \psi_1 \quad (7)$$

Equations (6) and (7) show that gearbox volume is a function of input drive volume V_{r1} . Weight of the whole gearbox is proportional to the sum of the volume of each gear in the gearbox. Based on that equation (6) can be used for pinion drive $d_2 = d_1 * i$, where d_2 – pitch diameter of pinion drive. Hence

$$b_1 * d_2^2 * \psi_2 = b_1 * d_1^2 * i^2 * \psi_1 \quad (8)$$

For offset gears (fig.1a) summary weight will be proportional to equation:

$$\sum_{i=1}^2 \psi_i * b_i * d_i^2 = \frac{2000 * T_1}{K} * \frac{i+1}{i} * (1 + i^2) \quad (9)$$

If $C = \frac{2000 * T_1}{K}$ then

$$M = \frac{\sum b_i * d_i^2 * \psi_i}{C} = \frac{1}{i} + 1 + i + i^2 \quad (10)$$

where $M = \frac{\sum b_i * d_i^2 * \psi_i}{C}$ is a gearbox mass factor. Equation

(10) allows to provide gearbox ratio with minimum mass of the gears.

Gearbox mass calculation methodology has also application to evaluate mass of the geartrain with idler gear (fig. 1b). Assuming that i_1 – ratio between input gear and idler gear, i_2 – ratio between idler and output gear, i_0 – overall ratio, then

$$i_0 = i_1 * i_2 \quad (11)$$

Pitch diameter of the idler gear is

$$d_2 = d_1 * i \quad (12)$$

equation (6) is

$$b * d_2^2 = 2000 * \frac{2 * T_1}{K} * \frac{i_1 + 1}{i_1} * i_1^2 \quad (13)$$

If it is taken under consideration that

$$d_3 = i_0 * d_1$$

where d_3 – pitch diameter of the output gear, then

$$b * d_3^2 = 2000 * \frac{2 * T_1}{K} * \frac{i_1 + 1}{i_1} * i_0^2 \quad (14)$$

Assuming that gears in the gearbox are not fully machined, we can get gearbox mass factor M

$$M = \frac{\sum_{i=1}^3 b_i * d_i^2 * \psi_i}{C} = 1 + \frac{1}{i_1} + i_1 + i_1^2 + i_0 + \frac{i_0^2}{i_1} \quad (15)$$

There is only one value of the ratio i_1 , for given ratio i_0 , at which function M is minimal, which corresponds to minimal mass of the given gearbox stage.

Calculating derivative of equation (15) and resulting equation is set equal to zero we get

$$\frac{dM}{di_1} = 1 + 2 * i_1 - \frac{1}{i_1^2} - \frac{i_0^2}{i_1^2} = 0 \quad (16)$$

and after next transformations

$$2 * i_1^3 + i_1^2 = i_0^2 + 1 \quad (17)$$

For given overall gearbox ratio i_0 , from equation (17), gearbox input ratio i_1 can be calculated for which mass will be minimal. For example at give $i_0=5$ input ratio $i_1=2.196$.

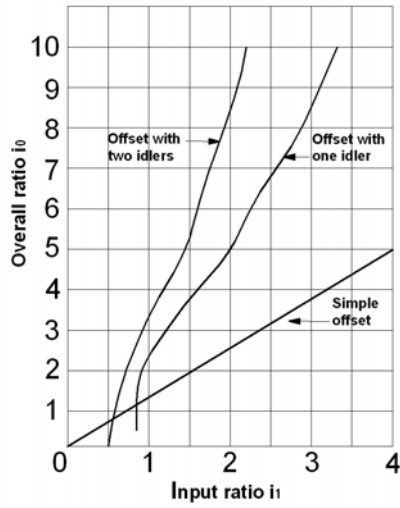
3. Results of discussion

Taking into consideration equation (15) it is possible to calculate mass function, which can be a baseline to compare different gears arrangement. To be able to estimate gearbox weight quickly it is necessary to describe presented procedure for different gearbox arrangements in graphical form or computer algorithm [5], which allow to take quick decision of kinematic drive scheme at minimum weight.

As an example of graphical representation mentioned methodology for gears arrangements in fig. 1a, b, c, can be weighted curves shown in figure 2. Based on fig. 2a it is possible to determine input ratio i_1 for given overall ratio i_0 . From figure 2b mass factor M can be determined in function of the given overall ratio i_0 .

The presented design methodology has also an application for other types of gearbox arrangements. Next it will be presented methodology for gears arrangements shown in figure 1 d, e, f.

a)



b)

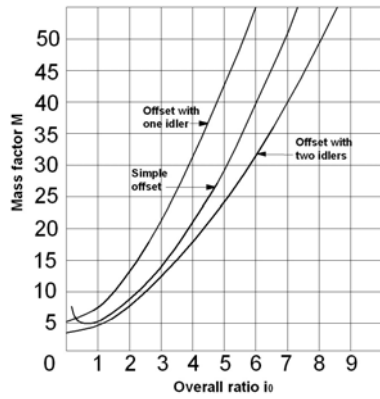
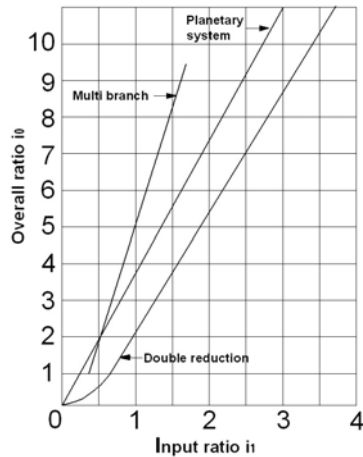


Fig. 2 Offset gears characteristics: a) minimum weight curves, b) total weight curves

Figure 3a shows relationship between overall ratio i_o and input ratio i_i for double reduction, multi branch and planetary gear system. Figure 3b allows to find mass factor M and what next, mass of the gearbox. For gears arrangements showed on figure 1 d, e, f minimum weight equations has been established, which allows to determine gearbox input ratio i_i for gearbox with minimum weight. Equations are presented in table 1.

a)



b)

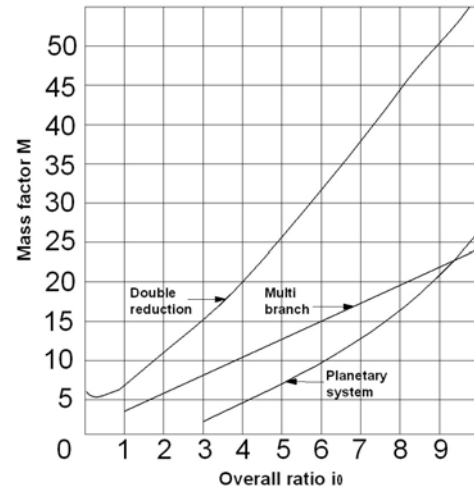


Fig. 3 Double reduction gears characteristics : a) minimum weight curves, b) total weight curves

Table 1: Minimum weight equations

Gearbox type	Minimum weight equation
Double reduction	$2 * i_1^3 + \frac{2 * i_1^2}{\left(\frac{i_0 + 1}{i_0}\right)} = \frac{i_0^2 + 1}{\left(\frac{i_0 + 1}{i_0}\right)}$
Multi branch (four branch)	$2 * i_1^3 + \frac{2 * i_1^2}{\left(\frac{i_0 + 1}{i_0}\right)} = \frac{i_0^2 + 1}{4 * \left(\frac{i_0 + 1}{i_0}\right)}$
Planetary	$2 * i_0^3 + i_0^2 = \frac{0.4 * (i_1 - 1)^2 + 1}{n}$

Where: n – number of planets, i_s – ratio between plane and sun gear.

Accordingly mass function M equations are (table 2):

Table 2: Total weight equations

Gearbox type	Total weight equation
Double reduction	$1 + \frac{1}{i_1} + 2 * i_1 + i_1^2 + \frac{i_1^2}{i_0} + \frac{i_0^2}{i_1} + i_0$
Multi branch (four branch)	$\frac{1}{4} + \frac{1}{4 * i_1} + 2 * i_1 + i_1^2 + \frac{i_1^2}{i_0} + \frac{i_0^2}{4 * i_1} + \frac{i_0}{4}$
Planetary	$\frac{1}{n} + \frac{1}{n * i_s} + i_s + i_s^2 + \frac{0.4 * (i_0 - 1)^2}{n * i_s} + \frac{0.4 * (i_0 - 1)^2}{n}$

4. Conclusions

Weight of the gearbox is the basic criteria used in aircraft industry. Gearbox arrangement at the early phase of the design has serious impact on the gearbox weight. Application of the gearbox ratio share methodology for specific gearbox stages allows to minimize gearbox weight.

5. References

- [1]. AGMA 911-A94 Design guidelines for aerospace gearing.
- [2]. Di Francesco G., Marini S.: Structural analysis of asymmetrical teeth: reduction of size and weight. Gear technology, September 1997, p.47-51.
- [3]. Dudley D.W.: Handbook of practical gear design. CRC Press, New York 2002.
- [4]. Müller L.: Przekładnie zębate. WNT Warszawa 1996.
- [5]. Rejman M., Rejman E.: Gearbox weight calculation spreadsheet. Rzeszów 2010 (not publicized).